Section A memo

Question 1

a) $x^3 + 2x^2 + 4x + 8 = 0$

$x^2(x+2) + 4(x+2) = 0$ ✓

$(x+2)(x^2+4) = 0$ ✓ (many other methods)

$x = -2$ ✓ or $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$ ✓✓

5

b) $xy = 5 + i$

$(a+bi)(c-i) = 5 + i$ ✓

$ac - ai + bci - bi^2 = 5 + i$

$ac + b - ai + bci = 5 + i$

$ac + b = 5$ ✓ $-a + bc = 1$ ✓

$\therefore -a + 3c = 1$

$x + y = 3 + 2i$

$a + bi + c - i = 3 + 2i$

$a + c = 3$ ✓ ; $b - 1 = 2$

$\therefore \underline{b = 3}$ ✓

$-a + 3c = 1$ ✓

$a + c = 3$

$4c = 4$ ✓

$\underline{c = 1}$ $\therefore a + 1 = 3$

$\underline{a = 2}$ ✓

8

13 marks

Question 2

$$\frac{x+2}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$$

$C: \frac{1}{1} = 1$

$$\therefore \frac{Ax(x+1) + B(x+1) + 1(x^2)}{x^2(x+1)}$$

$$\frac{Ax^2 + Ax + Bx + B + x^2}{x^2(x+1)}$$

$A + 1 = 0$ $\qquad$ $A + B = 1$

$\therefore A = -1$ $\qquad$ $B = 2$

7

$$\therefore \frac{x+2}{x^2(x+1)} = -\frac{1}{x} + \frac{2}{x^2} + \frac{1}{x+1}$$

7 marks

$$\text{OR} = \frac{-x+2}{x^2} + \frac{1}{x+1}$$

Question 3

a) $\dfrac{x^2+1}{3-x} \le x$

$\dfrac{x^2+1}{3-x} - x \le 0$

$\cancel{\dfrac{x^2+1}{3-x} - x}$

$\dfrac{x^2+1-x(3-x)}{3-x} \le 0$

$\dfrac{x^2-3x+x^2+1}{3-x} \le 0$

$\dfrac{2x^2-3x+1}{(3-x)} \le 0$

$\dfrac{(2x-1)(x-1)}{(3-x)} \le 0$

$+ \quad + \; 0 \quad - \; - \quad 0 \quad + \; + \quad U/D \quad - \; -$

$\frac{1}{2} \qquad 1 \qquad 3$

$\therefore \frac{1}{2} \le x \le 1$ or $x > 3$

8

b i) $\ln 36 = \ln 6^2 = 2\ln 6 = 2(\ln 3 + \ln 2)$

$= 2(1.10 + 0.693)$

$= 2 \times 1.793$

$= 3.586$

4

ii) $4\ln x - \frac{1}{2}\ln y + \ln z$

$= \ln x^4 - \ln\sqrt{y} + \ln z$

$= \ln \dfrac{x^4 . z}{\sqrt{y}}$

2

c). 1) TP: $x = 0$

$f(0) = \frac{1}{2}(e^0 + e^0)$

$= 1$ (2)

$\therefore$ minimum height above ground = 1m

2) Very dangerous to have an electricity cable hanging 1m above ground - people can walk into it. (2)

3) $2 = \frac{1}{2}(e^{\frac{d}{1}} + e^{-\frac{d}{1}})$

$4 = e^d + e^{-d}$

$4 = e^d + \frac{1}{e^d}$

$4e^d = e^{2d} + 1$

$e^{2d} - 4e^d + 1 = 0$

Let $e^d = k$

$k^2 - 4k + 1 = 0$

$k = \frac{-(-4) \pm \sqrt{16 - 4(1)(1)}}{2(1)}$

$= \frac{4 \pm \sqrt{12}}{2}$

$k = 3,732$

or $d = \ln 3,732....$

$\approx 1,32$

$k = 0,2679...$

$e^d = 0,2679...$

$d = -1.316$

$\approx -1,32$

(8)

$\therefore$ distance from origin is 1,32m

25 MARKS

Question 4

a) $f'(2) = -1$

$f'(0) =$

$f(x) = x^{1/3}$

$\therefore f'(x) = \frac{1}{3} x^{-2/3}$

$= \frac{1}{3\sqrt[3]{x^2}}$

(6)

$f'(0) = \frac{1}{3\sqrt[3]{0}}$ $\therefore$ $f'(0)$ does not exist.

b) $x = 1$ jump discontinuity (3)

c) $x = 0$ $\lim_{x \to 0^+} f'(x) \neq \lim_{x \to 0^-} f'(x)$ $\therefore$ not differentiable.

$x = 1$ Not differentiable because not continuous

(6)

$x = 3$ $\lim_{x \to 3^+} f'(x) \neq \lim_{x \to 3^-} f'(x)$

$\therefore$ Not differentiable

14 marks

Question 5

a) i) $\lim_{x \to \infty} \dfrac{5 - 3x - 2x^2}{4x^2 - 12x + 9}$ $\div$ thru by x^2

$= \lim_{x \to \infty} \dfrac{\frac{5}{x^2} - \frac{3}{x} - 2}{4 - \frac{12}{x} + \frac{9}{x^2}}$

$= \dfrac{-2}{4}$

$= -\frac{1}{2}$

5

ii) $\lim_{x \to 0} \dfrac{x^2}{\tan 2x \cdot \tan 3x}$

$= \lim_{x \to 0} \dfrac{2x}{2\tan 2x} \cdot \dfrac{3x}{3\tan 3x}$

5

$= \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$.

b) i) $y = \frac{5}{x}(1 + \sqrt{x})$

$= \frac{5}{x}(1 + x^{\frac{1}{2}})$

$= \frac{5}{x} + \frac{5}{x^{\frac{1}{2}}}$ multiply out

$= 5x^{-1} + 5x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -5x^{-2} + \frac{5}{2} x^{-\frac{3}{2}}$

$= \frac{-5}{x^2} + \frac{5}{2\sqrt{x^3}}$

5

ii) $y = \sqrt{x^2+1} + 4(3x+1)^5$

$y = (x^2+1)^{\frac{1}{2}} + 4(3x+1)^5$

$\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x + 20(3x+1)^4 \cdot 3$

$= \dfrac{x}{\sqrt{x^2+1}} + 60(3x+1)^4$

6

iii) $y = 3x \cdot \cos^3 2x$

$\frac{dy}{dx} = 3x \cdot 3\cos^2 2x \cdot (-\sin 2x) \cdot 2 + \cos^3 2x \cdot 3$

6

$= -18x \sin 2x \cdot \cos^2 2x + 3\cos^3 2x$.

c) i) $x^3 + y^3 = xy$

$3x^2 + 3y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$

$3y^2\frac{dy}{dx} - x\frac{dy}{dx} = y - 3x^2$

$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$

$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$

7

ii) At $(1,1)$ $\quad m = \frac{1 - 3(1)^2}{3(1)^2 - 1} = \frac{-2}{2} = -1$

eqn of tang $\quad y - 1 = -1(x - 1)$

$y - 1 = -x + 1$

$\underline{y = -x + 2}$

4

d) $f(x) = \frac{1}{2x^2}$

$= \frac{1}{2}x^{-2}$

$f'(x) = -x^{-3} \qquad = -1 \cdot x^{-3}$

$f''(x) = 3x^{-4} \qquad = -1 \cdot (-3)\, x^{-4}$

$f'''(x) = -12x^{-5} \qquad = -1 \cdot (-3)(-4)\, x^{-5}$ $\qquad 1, 3, 12, 60$

$f^{(4)}(x) = 60x^{-6} \qquad = -1 \cdot (-3)(-4)(-5)\, x^{-6}$

$f^n(x) = (-1)^n \frac{(n+1)!}{2} x^{-(n+2)}$

6

$\underline{\underline{44 \text{ marks}}}$

Question 6

a) $f(x) = -e^{x+1} + 2$

Cut on x-axis:

$0 = -e^{x+1} + 2$

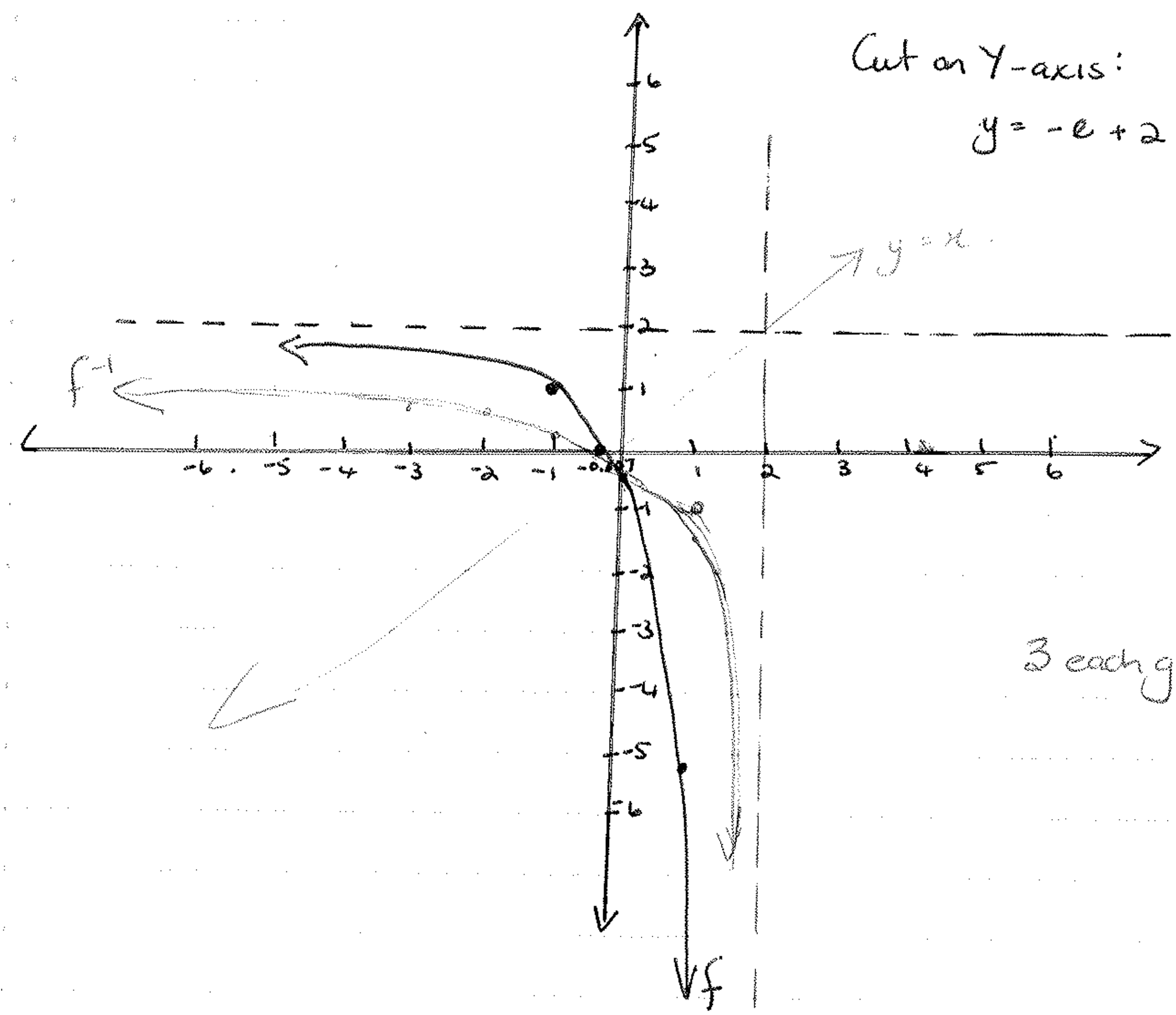
$e^{x+1} = 2$

$x+1 = \ln 2$

$x = \ln 2 - 1 = -0.307$

Cut on y-axis:

$y = -e + 2 \approx -0.3$



3 each graph.

b) $y = -e^{x+1} + 2$

Inv $x = -e^{y+1} + 2$

$x - 2 = -e^{y+1}$

$2 - x = e^{y+1}$

$\ln(2-x) = y+1$

$y = \ln(2-x) - 1$ $\therefore$ $f^{-1}(x) = \ln(2-x) - 1$

3

c) Domain: $x \in \mathbb{R}$

Range: $y < 2$

2

Question 7

a) x int. let $y=0$:

$$0 = \frac{x^2+2x+1}{x-1}$$

$$\therefore 0 = x^2+2x+1$$

$$0 = (x+1)(x+1)$$

$$\therefore x = -1$$

3

Y int let $x=0$:

$$y = \frac{1}{-1} = -1$$

2+

b) Vertical Asymptote : $x = 1$

2

Horizontal Asymptote :

$$y = \lim_{x\to\infty} \frac{x^2+2x+1}{x-1} \quad \div \text{ by } x^2$$

$$= \lim_{x\to\infty} \frac{1+\frac{2}{x}+\frac{1}{x}}{\frac{1}{x}-\frac{1}{x^2}}$$

Doesn't exist

$\therefore$ No horizontal asymptote.

Oblique. :

$$\begin{array}{r|l} & x + 3 \\ \hline x-1 & x^2+2x+1 \\ & x^2 - x \\ \hline & 3x+1 \\ & 3x-3 \\ \hline & 4 \end{array}$$

4

$\therefore$ Obique asymptote $y = x+3$

c)

$$f'(x) = \frac{(x-1)(2x+2) - (x^2+2x+1)(1)}{(x-1)^2}$$

5

$$= \frac{2\cancel{x^2}+\cancel{2x}-\cancel{2x}-2-\cancel{x^2}-2x-1}{(x-1)^2}$$

$$= \frac{x^2-2x-3}{(x-1)^2} = \frac{(x+1)(x-3)}{(x-1)^2} = \frac{(x+1)(x-3)}{(x-1)^2}.$$

d) Stationary pts : $f'(x) = 0$

$$\frac{(x+1)(x-3)}{(x-1)^2} = 0$$

$\therefore$ At $x = -1$ or $x = 3$.

$y = 0$ $\qquad$ $y = 8$.

$(-1;0)$ $\qquad$ $(3;8)$

$$f''(x) = \frac{(x-1)^2(2x-2) - (x^2-2x-3)2(x-1)}{(x-1)^4}$$

$$f''(-1) = \frac{4(-4) - (1+2-3)2(-2)}{(-2)^4} = \frac{-16+0}{16} = -ve$$

$\therefore$ local max.

$\therefore (-1,0)$ local max.

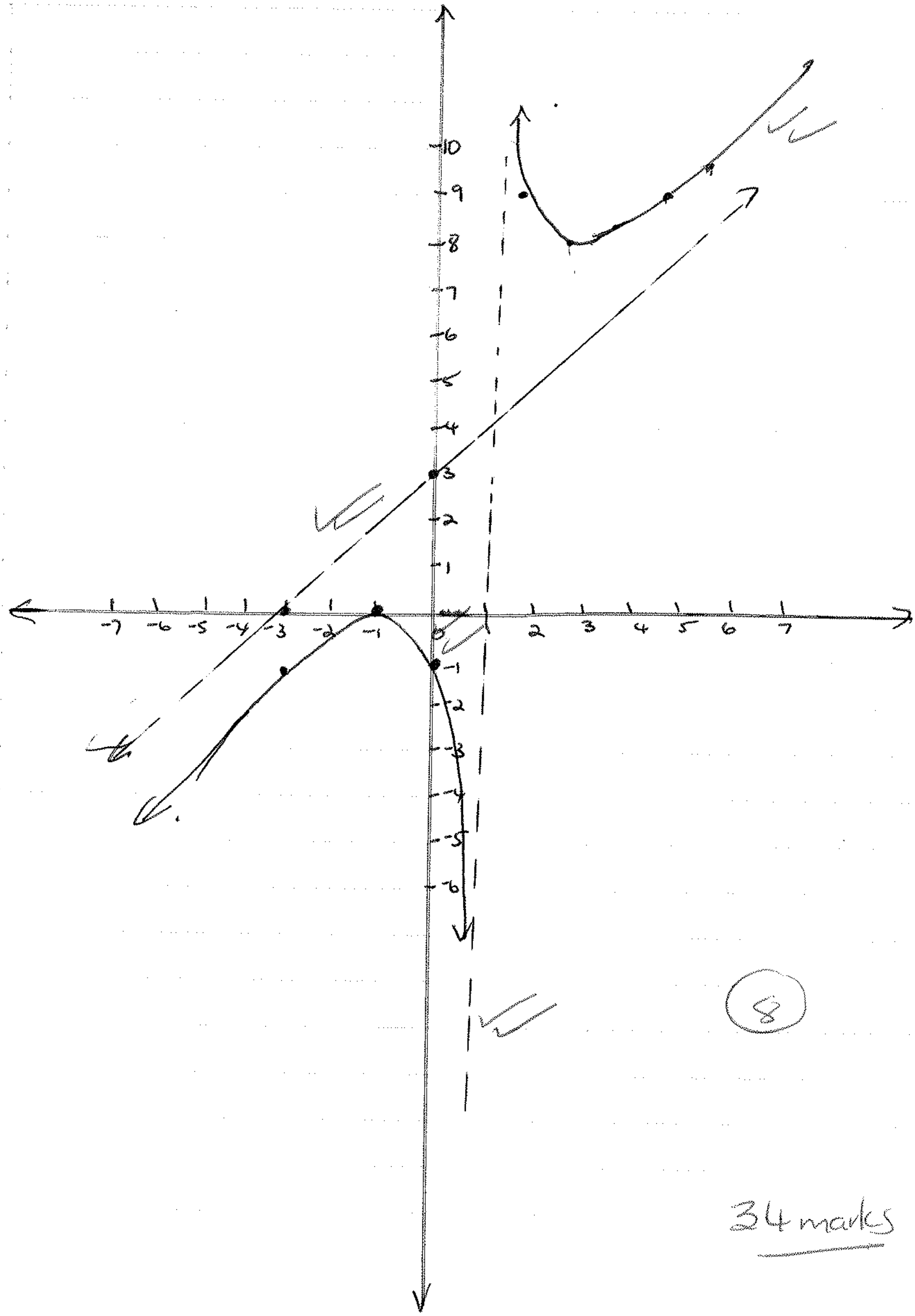
$(3;8)$ $\qquad f''(3) = \dfrac{4(4) - (9-6-3)2(3-1)}{(3-1)^4}$

$= \dfrac{16}{16}$

$= 1 > 0$ local min

$\therefore (3,8)$ local min.

10



8

34 marks

Question 8

a) $\int_{-1}^{2}\left(\frac{1}{x^3}+1\right)dx$

$= \int_{-1}^{2}\left(x^{-3}+1\right)\,dx$

$= \left[-\frac{1}{2}x^{-2}+x\right]_{-1}^{2}$

$= -\frac{1}{2}(2)^{-2}+2 - \left[-\frac{1}{2}(-1)^{-2}+(-1)\right]$

$= -\frac{1}{8}+2 - \left[-\frac{1}{2}-1\right]$

$= \frac{15}{8} + \frac{1}{2}+1$

$= \frac{15+4+8}{8}$

$= \frac{27}{8}$

6

b) $\int \frac{4x}{\sqrt{4-x^2}}\,dx$

Let $u = 4-x^2$

$\frac{du}{dx} = -2x\,dx$ $\qquad -\frac{1}{2}du = x\,dx$

$= \int \frac{4.\,-\frac{1}{2}du}{\sqrt{u}}$

$= \int -2u^{-\frac{1}{2}}\,du$

$= -4u^{\frac{1}{2}} + C$

8

$= -4(4-x^2)^{\frac{1}{2}} + C$

$= -4\sqrt{4-x^2} + C$

c) $\int \sin 4x \ \cos 3x \, dx$ $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$= \frac{1}{2}\int (\sin 7x + \sin x)\, dx$

$= \frac{1}{2}\left[-\frac{1}{7}\cos 7x - \cos x \right] + c$

$= -\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + C$

7

Question 9

$$DC^2 = r^2 + r^2 - 2r.r\cos\theta$$
$$= 2r^2 - 2r^2\cos\theta = 2r^2(1-\cos\theta)$$

a) Area shaded seg $= \frac{1}{2}r^2\theta - \frac{1}{2}r.r.\sin\theta$

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

Area of square

$= DC \times DC$

$= 2r^2(1-\cos\theta)$

$$8\left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right) = 2r^2(1-\cos\theta)$$

$$4r^2\theta - 4r^2\sin\theta = 2r^2 - 2r^2\cos\theta.$$

$$\div\ 2r^2.$$

$$2\theta - 2\sin\theta = 1 - \cos\theta$$
$$2\theta - 2\sin\theta + \cos\theta - 1 = 0$$

8

b) i) $2\theta - 2\sin\theta + \cos\theta - 1$

$= 2(1) - 2\sin 1 + \cos 1 - 1$

$= -0,1426$

2

ii) $2(2) - 2\sin 2 + \cos 2 - 1$

$= 0,7652$ ...

2

c) Guess 1.5

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$= x_r - \frac{2x_r - 2\sin x_r + \cos x_r - 1}{2 - 2\cos x_r - \sin x_r}$$

6

$x_1 = 1,5$

$x_2 = 1.412$

$x_3 = 1.4015$

$x_4 = 1.4013$

$x_5 = 1.40134$

Question 10

a) $TS^2 + 2^2 = x^2$

$TS^2 = x^2 - 4$

$TS = \sqrt{x^2-4}$

(2)

$SH = 6 - \sqrt{x^2-4}$ km

b) time is $\frac{\text{dist}}{\text{speed}}$

$t = \text{time} = \frac{x}{3} + \frac{6-\sqrt{x^2-4}}{5} = \frac{1}{3}x + \frac{6}{5} - \frac{1}{5}(x^2-4)^{1/2}$

min time will happen if $t' = 0$

$\frac{1}{3} - \frac{1}{10}(x^2-4)^{-1/2} \cdot 2x = 0$

$\frac{1}{3} - \frac{x}{5\sqrt{x^2-4}} = 0$

$5\sqrt{x^2-4} - 3x = 0$

$5\sqrt{x^2-4} = 3x$

$25(x^2-4) = 9x^2$

$25x^2 - 100 = 9x^2$

$16x^2 = 100$

$x^2 = \frac{100}{16}$

$x = \frac{10}{4}$

$= 2{,}5$ km

(10)

12 marks

SECTION B

Question 1

a) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

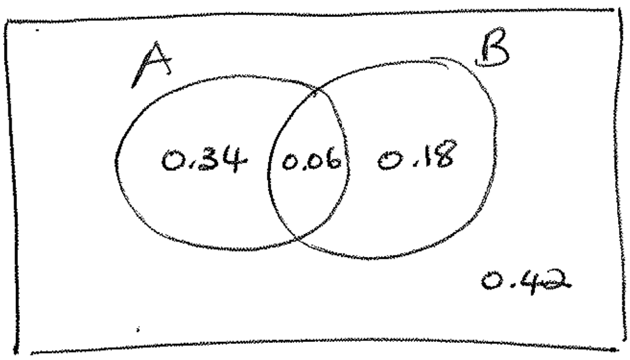
$0,25 = \dfrac{P(A \cap B)}{0.24}$ ✓

2

$P(A \cap B) = 0.24 \times 0.25$ ✓

$= \underline{0.06}$

b)



i) p(at least one event occurs) $= 1 - 0.42$ ✓ 2

$= \underline{0.58}$ ✓

ii) p(exactly one event occurs) $= 0.34 + 0.18$ ✓✓ 3

$= 0.52$ ✓

iii) $p(B|A) = \dfrac{p(A \cap B)}{p(A)}$ ✓ $= \dfrac{0.06}{0.4}$ ✓ $= 0.15$ ✓ 3

OR $\dfrac{0.06}{0.4}$ from Venn.

c) $p(A) \times p(B) = 0.4 \times 0.24 = 0.096$ ✓

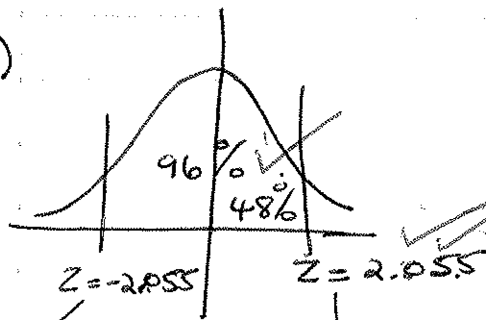
3

$p(A \cap B) = 0.06$ ✓

$\therefore p(A) \times p(B) \neq p(A \cap B)$ $\therefore$ events are <u>not</u> independent ✓

Question 2

a)



X = 　　　　X =

$$z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\frac{z\sigma}{\sqrt{n}} = x - \mu$$

$$\mu = x \pm \frac{z\sigma}{\sqrt{n}}.$$

$$\mu = 0.982 \pm 2.055 \times \frac{0.068}{\sqrt{20}}.$$

$$0,9508 \leq \mu \leq 1.0132\ l.$$

8

b) No mostly they will have slightly more or less than litre in a bottle but the avg amount of wine will be 1l.

2

c)

$$\frac{\binom{15}{1}\binom{10}{1}\binom{5}{1}}{\binom{30}{3}} = \frac{75}{406} = 0,1847$$

10

20 marks

Question 3

a) $\binom{12}{7}(0.4)^7(0.6)^5$

$= \underline{0.1009}$

7

b). $p = 1 - \dfrac{\binom{12}{0}\binom{78}{5}}{\binom{90}{5}}$

$= 1 - 0.48$

$= \underline{0.52}$

7

14 marks

Question 4

a) $\int_0^3 k(3x^2 - x^3) = 1$

$k\left[x^3 - \frac{1}{4}x^4\right]_0^3 = 1$

$k\left[27 - \frac{81}{4}\right] = 1$

$k = \frac{1}{\left[27 - \frac{81}{4}\right]}$

$= \frac{1}{\frac{108-81}{4}}$

$= \frac{4}{27}$

8

b) $f(x) = \frac{4}{27}(3x^2 - x^3)$

$= \frac{12}{27}x^2 - \frac{4}{27}x^3$

Mode will be when $f'(x) = 0$

$\frac{24}{27}x - \frac{12}{27}x^2 = 0$

6

$24x - 12x^2 = 0$

$12x(2 - x) = 0$

$x = 0$ or $x = 2$
N/A

$\therefore$ mode will be $x = 2$.

c) $\int_0^m \frac{4}{27}(3x^2 - x^3) = 0.5$

$\int_0^m (3x^2 - x^3) = \frac{27}{8}$

$\left[x^3 - \frac{1}{4}x^4\right]_0^m = \frac{27}{8}$

$m^3 - \frac{1}{4}m^4 = \frac{27}{8}$

$8m^3 - 2m^4 = 27$

5

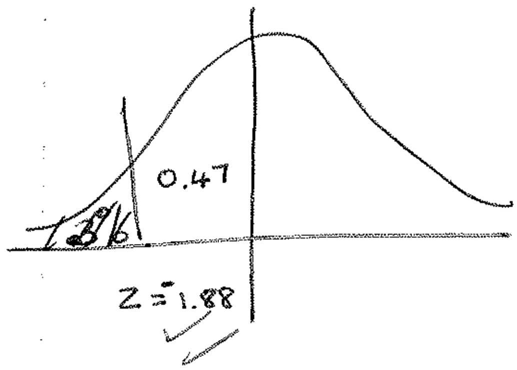
19 marks

Question 6

$\mu = 30$ $\qquad$ $\sigma^2 = 2,54$

$H_0: \quad \mu = 30$

$H_1: \quad \mu < 30.$



$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{28,8 - 30}{\frac{\sqrt{2,54}}{\sqrt{8}}}$$

$$= -2.1296$$

There is sufficient evidence to reject null hypothesis in favour of claim. Therefore there is cause for complaint.

12

Question 5

a) $\bar{x} = 66$

$\sum_1^{i} x_i = 990$

$\bar{x} = \frac{\sum_i x_i}{n}$

$66 = \frac{990}{n}$

$n = \frac{990}{66} = 15$

2

b) $\bar{y} = \frac{982}{15} = 65.47$

2

c) $b = \frac{n \sum_1^{i}(xy) - \sum x \sum_1 y}{n\left(\sum_1^{i} x^2\right) - \left(\sum_1^{i} x\right)^2}$ or $b = \frac{\sum_1^{i} xy - n\bar{x}\bar{y}}{\sum_1^{i} x^2 - n(\bar{x})^2}$

$= \frac{15(67680) - (990)(982)}{15(68900) - (990)^2}$

$= \underline{0,8056}$

6

$b = \frac{67680 - 15(66)(65.47)}{68900 - 15(66)^2}$

$= 0,8047$

because of rounding

without rounding $= 0.8056$

$y - \left(\frac{982}{15}\right) = 0.8056(x - 66)$

2

$\underline{y = 0.8056x + 12.2971}$

d) $y = 55{,}7995$ ②

e) $s_x^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \frac{68900}{15} - (66)^2 = \frac{712}{3}$

$\therefore s_x = 15{,}4056$

$s_y^2 = \frac{\sum y_i^2}{n} - \bar{y}^2 = \frac{67038}{15} - \left(\frac{982}{15}\right)^2$

$= 183.3315$

$s_y = 13{,}5394$

$\therefore r = b \times \frac{s_x}{s_y}$

$r = 0{,}8056 \times \frac{15.4056}{13{,}5394}$

④

$= \underline{0.9166}$

f) Very strong positive linear correlation. ②

g) It would be accurate because of the strong positive correlation coefficient. ②